

7. EXPONENTIAL SERIES AND LOGARITHMIC SERIES

Quick Review

1. If $x \in \mathbf{R}$ and $-1 < x \leq 1$ then $\log_e(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots$.
2. If $x \in \mathbf{R}$ and $|x| < 1$ then $\log_e(1-x) = -x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} - \dots$.
3. $\log 2 = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots$
4. (i) If $|x| < 1$ then $\log_e \frac{1+x}{1-x} = 2 \left(x + \frac{x^3}{3} + \frac{x^5}{5} + \dots \right)$.
(ii) If $x > 1$ then $\log_e \frac{x+1}{x-1} = 2 \left(\frac{1}{x} + \frac{1}{3x^3} + \frac{1}{5x^5} + \dots \right)$.
5. If $m, n \in \mathbf{N}$ and $m > n$ then $\log_e \frac{m}{n} = 2 \left[\left(\frac{m-n}{m+n} \right) + \frac{1}{3} \left(\frac{m-n}{m+n} \right)^3 + \frac{1}{5} \left(\frac{m-n}{m+n} \right)^5 + \dots \right]$